OMAR SHEMY PORTFOLIO

SUBJECT: Prestressing first principles study and flexural strength example

DATE	PROJECT NO.	PROJECT NAME	ENGINEER	PAGE
2024 JAN 19	**202403**	**DEMO PORTFOLIO**	**O.S.**	**1**

Prestressing general Notes:

Member Stress states

Initially in tension.

Initial Conditions: (top fibre)

$[P = P_i, \quad M_{min} = S_W]$

initial stresses in the top fibre (stresses)

$$-\frac{P}{A} + \frac{P.e}{S_t} - \frac{M}{S_t} \leq f_t \quad \begin{matrix} S.S\ ends \\ 0.5\sqrt{f'_{ci}} \end{matrix}$$

$$P\left(\frac{e}{S_t} - \frac{1}{A}\right) \leq f_t + \frac{M}{S_t}$$

While $k_b = \frac{S_t}{A} \rightarrow A = \frac{S_t}{k_b} \rightarrow \frac{1}{A} = \frac{k_b}{S_t}$

$$P\left(\frac{e}{S_t} - \frac{k_b}{S_t}\right) \leq f_t + \frac{M}{S_t}$$

$$P(e - k_b) \leq f_t S_t + M$$

$$P \leq \frac{f_t S_t + M}{e - k_b}$$

$$P_i \leq \frac{f_t S_t + M_{min}}{e - k_b}$$

P(10.23)

* Serviceability stress limit not Modulus of rupture.

$$f_t \begin{cases} 0.25\sqrt{f'_{ci}} & S_{in} \\ 0.5\sqrt{f'_{ci}} & S.S\ ends \end{cases}$$

$$S_t = \frac{I}{y_{top}}$$

Finally in tension

final Conditions (bottom fibre)

$[P = P_f, \quad M_{max} = \text{factored loads}]$.

$$-\frac{P}{A} - \frac{P.e}{S_b} + \frac{M}{S_b} \leq f_b$$

$$-P\left(\frac{1}{A} + \frac{e}{S_b}\right) \leq f_b - \frac{M}{S_b}$$

$$P\left(\frac{1}{A} + \frac{e}{S_b}\right) \geq -f_b + \frac{M}{S_b}$$

while $k_t = \frac{S_b}{A} \rightarrow A = \frac{S_b}{k_t} \rightarrow \frac{1}{A} = \frac{k_t}{S_b}$

$$P\left(\frac{e}{S_b} + \frac{k_t}{S_b}\right) \geq \frac{M}{S_b} - f_b$$

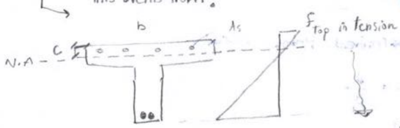
$$P(e + k_t) \geq M - f_b S_b$$

$$P_f \geq \frac{M_{max} - f_b S_b}{(e + k_t)}$$

$$f_b \begin{cases} 0.5\sqrt{f'_c} \\ 0.25\sqrt{f'_c} & \text{Corrosive environment} \end{cases}$$

$$S_b = \frac{I}{y_{bot}}$$

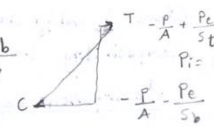
This stems from:



$f_{top} > 0.5\sqrt{f_{ci}}$

Place reinforcing bars with $A_s = \frac{\frac{1}{2} f_{top}\, c\, b}{0.5\, f_y}$

Schematics to Calculate f_{top}



$P_i = A_p f_{pi}$

Presstressing (Tendon) Stress limits P(10-23):

i.e. $f_{py} = 0.9\, f_{pu}$

	f_{py}	at Jacking	at transfer
Low Relaxation type	0.9	0.8	0.74

Concrete stress limits P(10-23):

	Compressive stress limits	Tensile stress limit
Initial Stage	$0.6\, f'_{ci}$	$0.25\,\lambda\sqrt{f_{ci}}$ $0.5\,\lambda\sqrt{f_{ci}}$ at simply supported ends
Final Stage	$0.45\, f'_{c}$	$0.5\,\lambda\sqrt{f'_c}$ $0.25\sqrt{f'_c}$ (if member exposed to corrosive environment).

Table 6.3 P(10 [Values for Precast Pretensioned members]

Low Relaxation type $f_{pu} = 1860$ MPa	f_p bed	f_{p_i} (initial)	$f_{p_{e,F}}$ (end final)
	$0.75\, f_{pu}$	1290	1080

Flexural resistance:

18.6.2:

(a) $f_{pr} = f_{pu}\left(1 - k_p \frac{c}{d_p}\right)$

$k_p = 2(1.04 - f_{py}/f_{pu})$

requirements: $\begin{cases} \text{bonded tendons} \\ c/d_p < 0.5 \\ \frac{f_{pe} \approx f_{ps}}{f_{py}} \geq 0.6 \end{cases}$

18.6.3 use $\phi_s A_s f_y$ and $\phi_s' A_s' f_y$ Provided they are located at least $0.75\,c$ from the N.A

18.7 at every section of a flexural member $M_r \geq 1.2 M_{cr}$

where $M_{cr} = \frac{1}{y_t}(f_{ce} + f_r)$

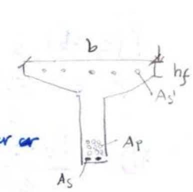
$y_t =$

$f_{ce} =$

Modulus of rupture $f_r = 0.6\sqrt{f'_c}$

$\Sigma C = \Sigma T$ for a T beam

$$\frac{c}{d_p} = \frac{\phi_p f_{pu} A_p + \phi_s f_y A_s - \phi_s f_y' A_s' - \alpha_1 \phi_c f'_c (b - b_w) h_f}{\alpha_1 \phi_c f'_c \beta_1 b_w d_p + k_p \phi_p f_{pu} A_p}$$



variations $A_s = A_s' = 0$ or $b = b_w$ $h_f > \beta_1 c$ Consider as a rectangular beam of width (b).

$$a = \frac{\phi_p A_p f_{pr} + \phi_s A_s f_s - \phi_s A_s f_s'}{\alpha_1 \phi_c f'_c b}$$

Where f_s and f_s' are the reinforcing bar stresses determined from strain Compatibility for a neutral axis depth of $c = \frac{a}{\beta_1}$

Development length of pretensioned strand

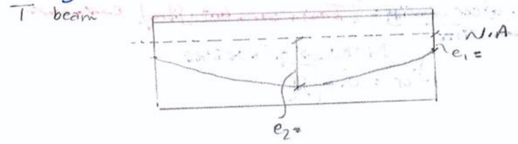
$l_d = 0.145(f_{pr} - 0.67 f_{pe}) d_b$

$l_d = 0.048 f_{pe} d_b + 0.145(f_{pr} - f_{pe}) d_b$

Add Guide:

Example techniques + Notes:

Calculating Stress Service Limit states (T-beam)



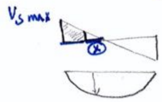
Check tensile stress service limit states are satisfied @ 40% along span:

given + A_{gross} * y_{big} = Centroid to bottommost fibre * I_{gross} * S_b = Section Modulus bottom

b = width of Top Flange

$S_b = \frac{I}{y_{bot}}$ ← Centroid to bottom fibre

So $w_S = \underset{\uparrow \text{kPa}}{bD} + \underset{\uparrow \text{kPa}}{bL} + \underset{\downarrow \text{mm}^2}{A_{gross}} * 24\frac{kN}{m^3} = \frac{kN}{m}$



$V_{s\,max} = \frac{w_s * L}{2}$

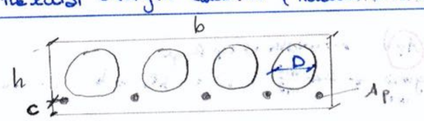
$\frac{V_{s\,max}}{L/2} = \frac{V_{s\,0.4l}}{x}$ Moments = Area under shear.

Since this is a $\boxed{\text{final stage}}$ stress limit, then bottom fibre is under tension in a corrosive environment so limit is $0.25\lambda\sqrt{f'_c}$

Service limit $f_b = -\frac{P_f}{A} - \frac{P_f\, e}{S_b} + \frac{M_{s\,max}}{S_b} =$ Should be $< 0.25\lambda\sqrt{f'_c}$

e @ $0.4\,l$

Example of Flexural strength calculations (hollow core SLAB):



\# of holes

S_b = given A_g = given precise or approximated as $bh - \frac{\pi D^2}{4}(\#)$

α_1 = and β_1 Calculated $\boxed{\phi_c = 0.7}$ Certified Precast plant

$A_p = \begin{cases} \# * 55\,mm^2 \text{ if } 9\,mm \\ \text{or} \end{cases}$

$d_p = h - c$

Determine $k_p = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right)$ → 0.9 for LOW RELAXATION STRAND

For the Rectangular section

$$\frac{c}{d_p} = \frac{\phi_p A_p f_{pu} + \phi_s A_s f_y - \phi_s A_s f_y}{\alpha_1 \phi_c f'_c \beta_1 b d_p + k_p \phi_p A_p f_{pu}} < 0.5$$

1) $f_{pr} = f_{pu}\left(1 - k_p \frac{c}{d_p}\right)$

$c = \left(\frac{c}{d_p}\right) * d_p$ and $a = \beta_1 c$ check $a <$ the depth of the top flange.

2) $M_r = \phi_p A_p f_{pr}\left(d_p - \frac{a}{2}\right)$

3) check Adequate reserve of Strength after cracking : (Tension on the bottom fibre). Extra strength not stress limit state like a Moment extra check.

$e = \frac{h}{2} - c$

Low relaxation

$f_{pe} = 1880$

$P_f = A_p * f_{pe}$

no ϕ factor

MODULUS OF RUPTURE

$f_{cr} = 0.6\sqrt{f'_c}$

Conditions for flexural cracking on the bottom can be expressed as:

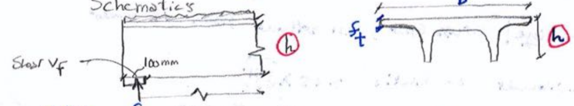
$$-\frac{P_f}{A} - \frac{P_f \cdot e}{S_b} + \frac{M_{cr}}{S_b} = f_{cr}$$

Solve for cracking Moment (M_{cr}).

Check $\frac{M_r}{M_{cr}} > 1.2$

Example Design of Double-Tee member

Schematics



given $f_{ci} =$ Stress at transfer

$f'_c =$ minimum Specified 28-day strength.

Steel type { Low; 9 mm or 13 mm }

$f_{pu} =$ tensioned to $f_{p\,bed} = 0.75 f_{pu}$

1) depth Choice check $\frac{h}{e}$ ratio (10-41).

2) Choice of prestressing (estimate A_p) $b =$ width of top flange

Loads = $S_w = A_{gross\,(mm^2)} * 24 \frac{kN}{m^3}$ $D = kPa * b$ $L = kPa * b$

all in (kN/m)

Maximum Service Moment at midspan =

$$M_{s\,max} = (S_w + D + L)\frac{L^2}{8}$$

Maximum factored Moment at Midspan:

$$M_{F\,max} = \left[1.25(D + S_w) + 1.5(L)\right]\frac{L^2}{8}$$

from (10-43) estimate $(e + k_t) \simeq 0.70h$,

CASE: DOUBLET

to satisfy final stress limit state on tension in the bottom fibre $f_b = 0.5\lambda\sqrt{f'_c}$

final f'_c

not corrosive

$$P_f \geq \frac{M_{s\,max} - S_b f_b}{e + k_t}$$

← Preliminary estimate

at this point we don't eccentricity yet in case of checking the design

e is obtained from the strand profile and $k_t = \frac{S_b}{A}$ precisely.

P_f estimate in kN

Since this is P_f then f_{pf} not f_{pu}. final stress in low relaxation strand after losses is (i.e. 1080).

$$A_{ps} \geq \frac{P_f \times 10^3}{f_{pf}} \quad \frac{kN}{N/mm^2}$$

to estimate A_{ps} required for flexure (prelim).

$$M_{F\,max} = M_r = 0.77\, A_{ps}\, f_{pu}\, h$$

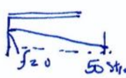
Ultimate stress ↩ ↑ Section depth

workout $\underline{A_{ps}}$.

Try to place equal number of strands in Double tee legs.

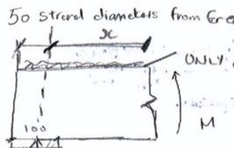
for 13-mm strands $A = 99\,mm^2$

3) tendon profile midspan (or) use maximum permissable but at the ends limit tensile stress on top face of beam.



$f_t = 0.5\,\lambda\sqrt{f'_{ci}}$ (at simply supported ends).

Investigate the stress conditions 50 strand diameters from this end face



Service Selfweight Moment

$$e \leq k_b + \frac{M_{min} + f_t S_t}{P_i}$$

$f_t = 0.5\,\lambda\sqrt{f'_{ci}}$

$P_i = A_{ps} \times f_{pi}$

no ϕ factor

S_t = top Section Modulus

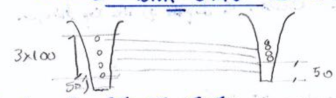
then find e.

$\underline{M_{min}}_{SW} = \left[\frac{wLx}{2} - wx^2\right]$ UNFACTORED

SELFWEIGHT MOMENT

Check or form strand Patterns

example



Calculate $\bar{y}$ of the strands. $\bar{y} = \frac{350+250+150+50}{4} = 200.$

$e_e = \text{Centroid} - \bar{y} < e$.

$\bar{y} = \frac{89+76+63+50}{4} = 70$

$e_c = \text{Centroid} - \bar{y}$

You want large e at the center.

Preliminary design Completed

4) Check Stresses @ SERVICE LOADS

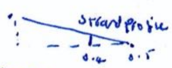
3 Locations two stress states { initial Condition, SW only, Prestress high; final condition, full service loads, Prestress losses occurred

[Midspan, @ 0.4 l, 50 Strand from end of the member].

Tabulate results ↙ all unfactored.

Initial stress limits	Stress elements	Locations: 50 strand diameters Top	bottom	at 0.4l Top	bottom	at midspan Top	bottom
tensil limit at ends $0.5\lambda\sqrt{f'_{ci}}$; $0.25\lambda\sqrt{f'_{ci}}$	$-\frac{P_i}{A}$						
Compressive limit $0.6\ f'_{ci}$	$\pm\frac{P_i \cdot e}{S}$						
	$\pm\frac{M_{min}}{S}$						
Final stress limits							
tensile limit non corrosive $0.5\lambda\sqrt{f'_c}$	$-\frac{P_f}{A}$						
Compressive limit $0.45\ f'_c$	$\pm\frac{P_f \cdot e}{S}$						
	$\pm\frac{M_{max}}{S}$						

5). Check flexural Capacities: Check @ 0.5 L and @ 0.4 l



Find d_p @ 0.5 at the midspan $d_p = h - y_{strand}$ (max compression fibre to y)

d_p @ 0.4l by interpolation

Calculate k_p

$$\frac{c}{d_p} = \frac{\phi_p A_p f_{pu}}{\alpha_1 \phi_c f'_c \beta_1 b d_p + k_p \phi_p A_p f_{pu}}$$

$$f_{pr} = f_{pu}\left(1 - k_p \frac{c}{d_p}\right)$$

$C = \left(\frac{c}{d_p}\right) * d_p$ and $a = \beta_1 c$ Check $a <$ depth of the top flange.

$$M_r = \phi_p A_p f_{pr}\left(d_p - \frac{a}{2}\right)$$

Repeat for 0.4 l

6).

Check adequate reserve of strength after cracking

$f_r = 0.6\sqrt{f'_c}$

$$-\frac{P_f}{A} - \frac{P_f e}{S_b} + \frac{M_{cr}}{S_b} = f_r$$

Calculate M_{cr}

Check $\frac{M_r}{M_{cr}} > 1.2$.